REPORT No. 388

INVESTIGATION OF THE DIAPHRAGM-TYPE PRESSURE CELL

By Theodore Theodorsen

SUMMARY

This report relates to various improvements in the process of manufacture of the N. A. C. A. standard pressure cell. Like most pressure recording devices employing thin diaphragms, they would in general show considerable change in calibration with temperature and also some change of calibration with time or aging effect. Some instruments exhibited considerable internal friction.

It was established that the temperature dependency of the stiffness was due to difference in the thermal expansivity between the diaphragm proper and the supporting body of the cell, and convenient methods for its compensation have been developed. The diaphragm is furnished with a small central bushing of a different metal, and it is possible to determine a size of this bushing which gives the diaphragm exactly the same thermal expansivity as the cell body.

It was further established that the internal hysteresis in the diaphragm was of a negligible magnitude and that the observed lag was due, primarily, to the force of the hairspring on the stylus point. The resultant adoption of weaker hairsprings made it possible to extend the useful range of the instrument considerably downward. Satisfactory instruments having a range of less than 3 inches of water were made possible.

It was found that the tendency to change calibration with time was caused, to a great extent, by insufficient clamping of the diaphragm. The adoption of double copper gaskets improved this condition.

The required diaphragm thickness and the desirable rate of mechanical magnification have been determined on the basis of several hundred tests.

INTRODUCTION

This report was prepared by the National Advisory Committee for Aeronautics. It gives the results of a systematic investigation undertaken at the Langley Memorial Aeronautical Laboratory during the fall of 1929. The investigation is rather general in its scope. The actual experimental work is, however, confined to tests on the N. A. C. A. standard instruments. These instruments, developed by the technical staff of the committee, are of a very simple and rugged design. The pressure cell is shown in Figure 1. It consists, essentially, of a flat, circular diaphragm or membrane, A, tightly stretched and securely clamped along its circumference. The unsupported diameter of the diaphragm is approximately 1⅛ inches, and the thickness is of the order of 0.001 inch to 0.006 inch. The motion of the diaphragm is transmitted by a small steel pin or stylus, E, to a rotatable mirror, F. The distance between the axis of the stylus and the axis of the mirror shaft is from 0.010 inch to 0.050 inch, approximately. For the present purpose it will be sufficient to state that the actual deflection of the diaphragm is reproduced on a greatly magnified scale by means of a beam of light reflected from the mirror. The maximum magnification employed is somewhat in excess of 1,000.

The light beam is arranged so as to form a sharp image of the source on a graduated scale for direct

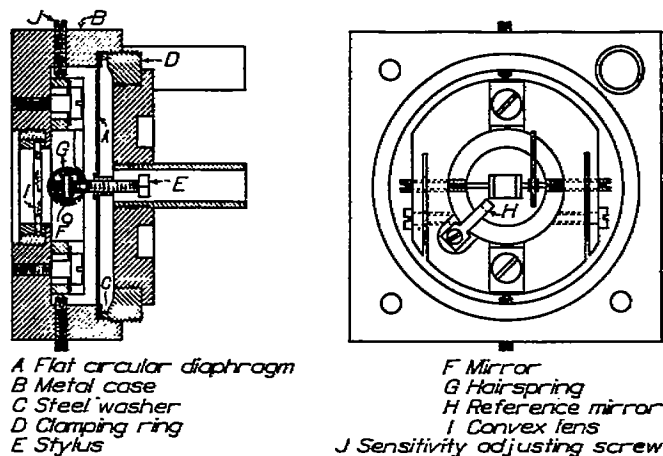|	
A Flat circular diaphragm	F Mirror
B Metal case	G Hairspring
C Steel washer	H Reference mirror
D Clamping ring	I Convex lens
E Stylus	J Sensitivity adjusting screw

FIGURE 1.—N. A. C. A. pressure cell

observation, or on a revolving film for recording purposes. The pressure cell carries a small stationary reference mirror, H. (See fig. 1.) For a more complete description of the instrument, see N. A. C. A. Technical Note No. 64 by F. H. Norton. (Reference 1.)

Because the diaphragm instrument is almost entirely free from mass effects, it is indispensable as a pressure-measuring device in all investigations performed on airplanes in flight. The acceleration in violent maneuvers may, at times, amount to more than 10 g. If this acceleration happens to be perpendicular to the diaphragm, it is equivalent to a pressure of approximately 0.2 inch of water on the most frequently employed diaphragm of 0.002 inch thickness. As the range of this diaphragm is about 20 inches of water, the above error is seen to amount to but one per cent of the full-

scale deflection. This error may, however, easily be compensated for by counterweights on the mirror staff, if necessary.

Contrasted to this and to other desirable qualities, it is also well known that instruments employing thin diaphragms as an integral element very often are subject to a number of peculiar effects of quite an obscure nature. In fact, the behavior of some diaphragms is so erratic that one may, at a first glance,

(1) Compensation of temperature effects.

(2) Removal of frictional effects.

It was known that the pressure cells usually showed a change in reading with temperature. The effect was very pronounced. Figures 2, 3, 4, 5, and 6 give an impression of the situation at the time the following investigation was started. The temperature effect amounts to as much as 30 per cent of the scale range. The calibration curves taken at different temperatures
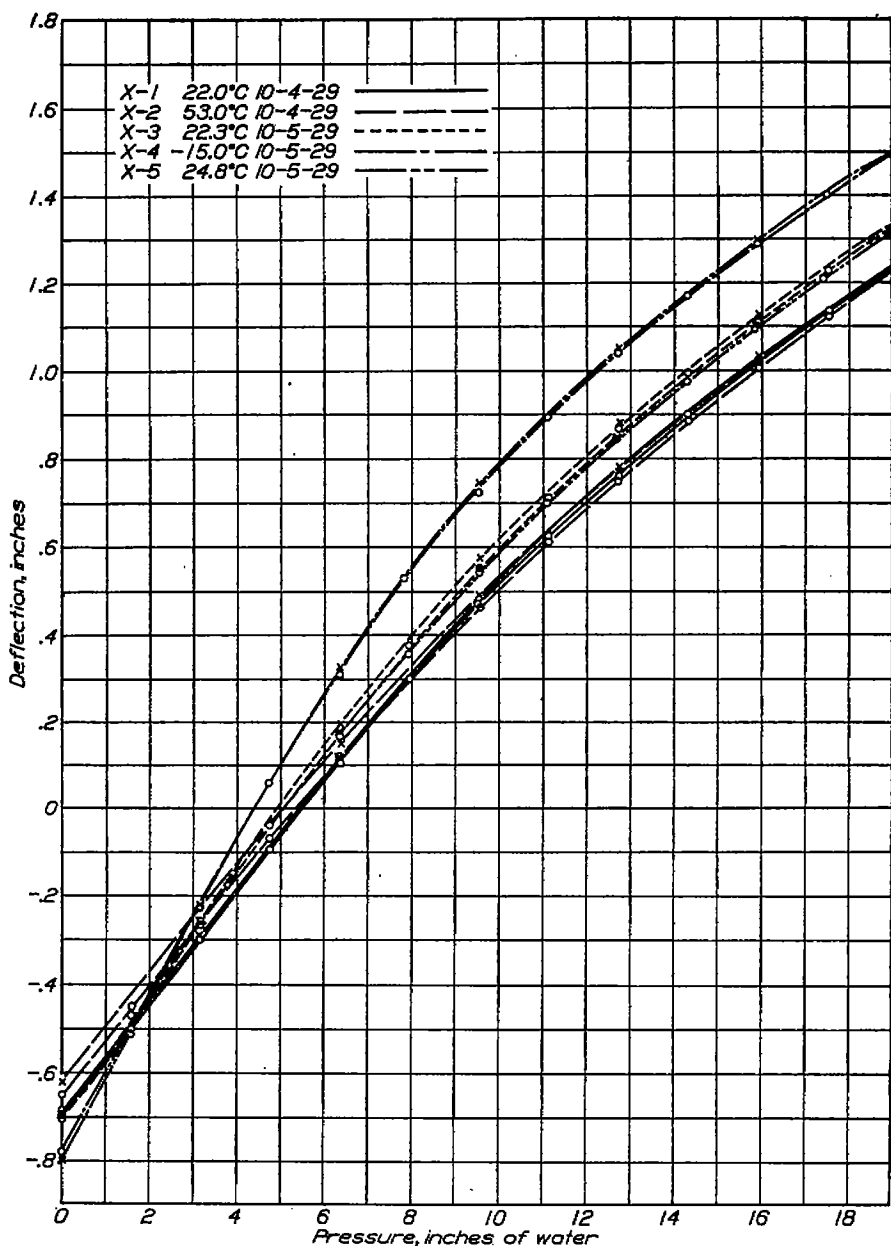

X-1	22.0°C	10-4-29	
X-2	53.0°C	10-4-29	
X-3	22.3°C	10-5-29	
X-4	-15.0°C	10-5-29	
X-5	24.8°C	10-5-29	

FIGURE 2.—Air-speed calibration. Instrument No. 136. Cell No. 230. Diaphragm 0.002". Steel bushing. Cell cooled to 0° F. Diaphragm stretched

be tempted to believe that they are subject to no laws at all.

The purpose of the following study was to produce such information as would increase the general knowledge regarding the behavior of the diaphragm and the predictability of its performance. The study was, for practical reasons, directly focused on the two following problems:

do not, in general, intersect at zero, as might be expected, but at some quite different pressure. The zero point is, consequently, subject to a change with temperature. (See figs. 13 and 14.) In some cases the curves do not intersect at all (see fig. 16), or they intersect beyond the range of the calibration. (See figs. 15 and 17.) The trouble increases rapidly with the sensitivity of the cell.

In this paper an attempt is made to analyze the causes of this phenomenon. It will be indicated to what extent the prediction and prevention of the condition is possible.

Even more troublesome than the temperature effect is the internal friction in the cells. The author has gone to great detail in tracing the origin of the lag,

nature will be more or less touched upon. Notwithstanding the very simple design of the cell, it was found to furnish quite a rich field for investigation.

METHODS OF CALIBRATION

The instruments are usually allowed to age for at least one day. They are then put through individual
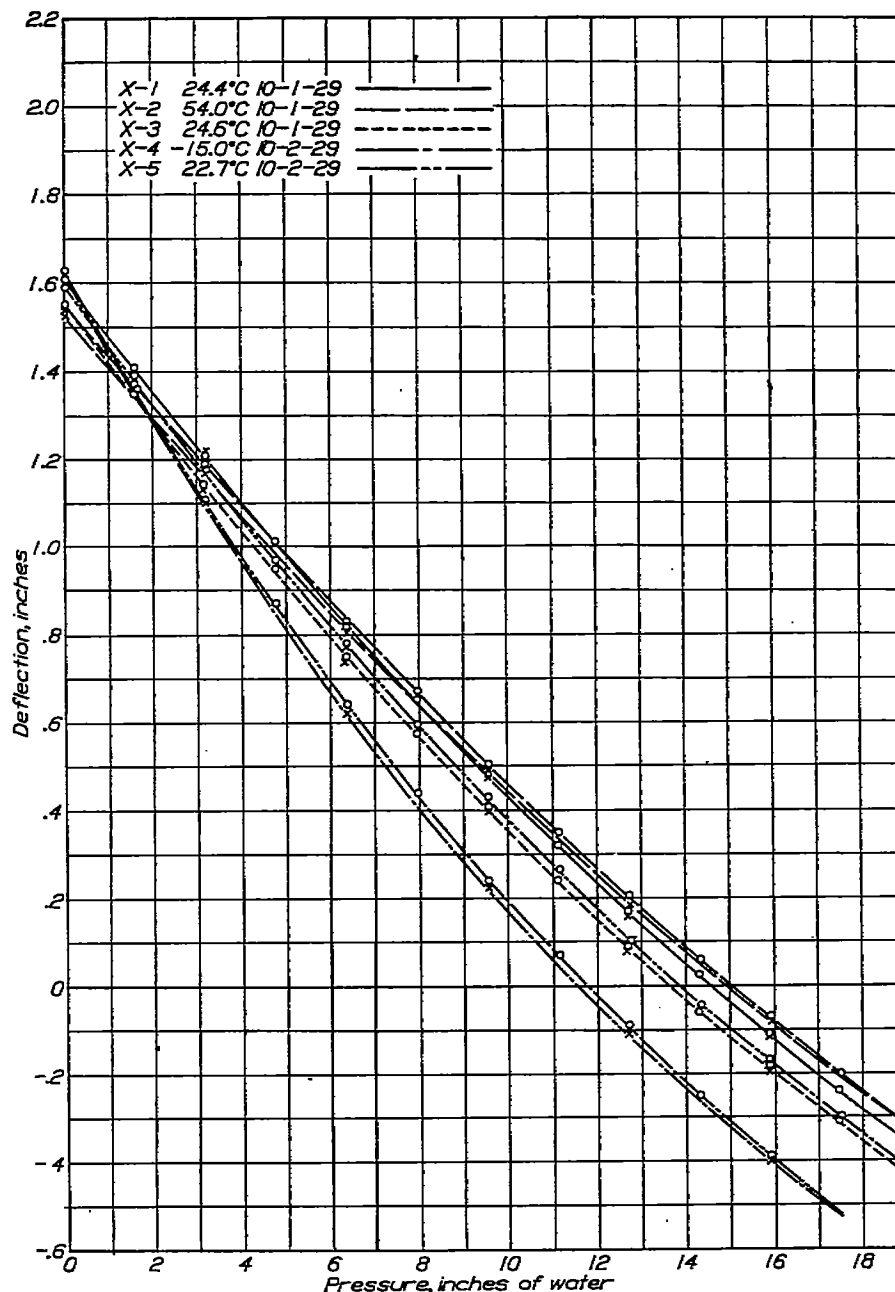

FIGURE 3.—Air-speed calibration. Instrument No. 149. Capsule No. G-160 (lower)

or dead motion, in the N. A. C. A. instrument. This effect is, however, only of importance for very sensitive cells, instruments having a full range of less than about 3 inches of water. The sources of the internal friction were analyzed. As a result, the most desirable condition could be definitely established, resulting in considerable improvement.

A number of related problems which, naturally, arise in connection with an investigation of this

calibrations at three different temperatures. The calibrations are taken in the following succession:

(1) Room temperature.
(2) − 15° C.
(3) Room temperature.
(4) + 53° C.
(5) Room temperature.

The instrument is kept at each desired temperature for one-half hour prior to exposure to insure an even

and correct temperature. In each case readings are taken both for increasing and for decreasing pressure over the full range. The deflections are recorded photographically on a nondistorting film. This procedure has been followed in the present investigation.

THEORY OF DIAPHRAGMS

It was soon established that most of the difficulties mentioned above could be traced back to the dia-

where p is the uniform pressure.

$\quad t$ is the thickness,

$\quad r$ is the radius,

$\quad W$ is the deflection at the center,

$\quad E$ is Young's Modulus of Elasticity,

$\quad \delta$ is Poisson's Ratio.

It will be noticed, in particular, that the only constant of the material appearing in the expression is the
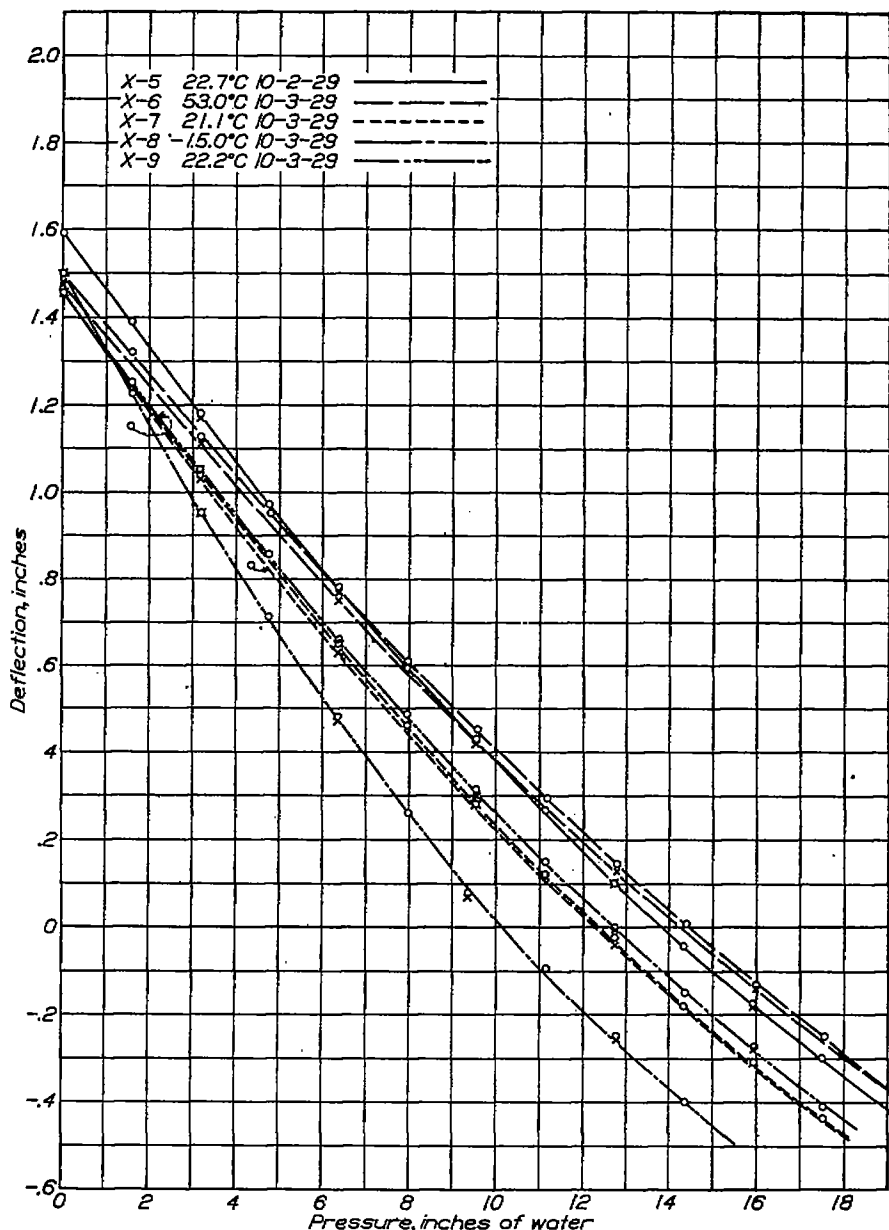

FIGURE 4.—Air-speed calibration. Instrument No. 149. Capsule No. G-160 (lower)

phragm itself. Unfortunately, there does not, at the present time, exist any general theory of stretched circular diaphragms. We are, however, able to draw a number of conclusions from particular cases for which the solutions are known.

The classical theory of thin diaphragms gives for the case of a circular plane diaphragm clamped at the circumference:

$$p = \frac{16}{3} \frac{E}{1-\delta^2} t^3 \frac{W}{r^4} \text{ (Reference 2)} \qquad (1)$$

so-called plate modulus $\frac{E}{1-\delta^2}$. It seems, then, quite consistent to draw the conclusion that any observed temperature dependency of the characteristic curves is, necessarily, caused by changes in the plate modulus with temperature.[1] This latter variability is, in turn, attributed to "cold working of the diaphragm, or to defective elasticity." (See for instance Journal of the

[1] There is a slight dependency brought about by changes in linear dimensions of the diaphragm with temperature. The magnitude is negligible.

Royal Aeronautical Society, Number 210, Volume XXXII, June, 1928, p. 444. Reference 3.)

A rather interesting study, relating directly to this question, is given in the N. A. C. A. Technical Report No. 165, by M. D. Hersey. (See paragraphs 21–22, on temperature compensation. Reference 4.) We quote from this paper:

departure in mechanical design, such as to satisfy equation (15) for any given values of α, β, and γ."

Mr. Hersey's discussion is, however, restricted to geometrically similar cases, which fact is clearly pointed out. In order to satisfy this requirement, in the case of a rigidly clamped diaphragm, when the temperature is subject to change, it is necessary that the diaphragm
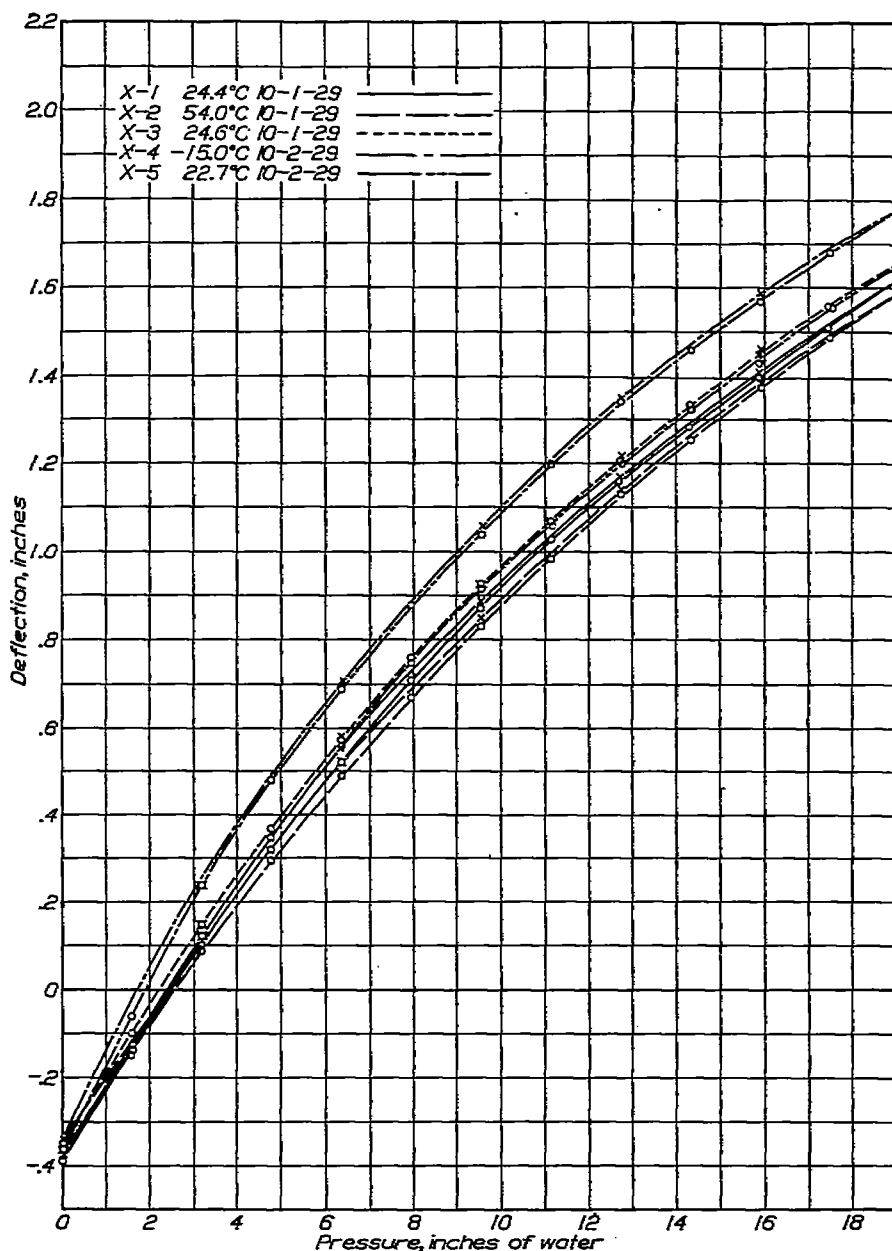

FIGURE 5.—Air-speed calibration. Instrument No. 149. Capsule No. G-280 (upper)

"The question whether intrinsic compensation is practicable remains as an important one for future study, but the conditions to be satisfied have been analyzed above. It is conceivable that compensation might be secured either by discovering alloys which satisfy equation (15) [2] for a given value of C; or conversely by developing a suitable value of C through some radical

and the casing have the *same coefficient of thermal expansion*. All that needs to be done in order to secure temperature compensation of any effects of the plate modulus is to *violate* this restriction of geometrical similarity with respect to temperature.

It is also quite conceivable that the large temperature coefficients which are sometimes observed are due to departure from similarity, caused by lack of homogeneity with respect to thermal expansion, rather than

[2] For the sake of brevity, it will just be stated that equation (15), containing the symbols C, α, β, and γ, essentially, gives the condition of a *constant plate modulus*.

by artificial conditions presumably affecting the moduli of elasticity.

Mr. Hersey states, for instance, that the coefficient of stiffness for a soft iron disk, supported freely on a sharp, brass ring, amounts to −6 per cent per degree centigrade, or, in other words, that the stiffness is doubled for every 16 degrees centigrade. A large

first place, the formula is correct for *small* deflections only, so as to secure proportionality between stresses and strains, as required. This fact is, however, no essential limitation on its general value. We have, in most cases, a straight-line relation between pressure and central deflection. Moreover, we are primarily interested in utilizing the formula in predicting the
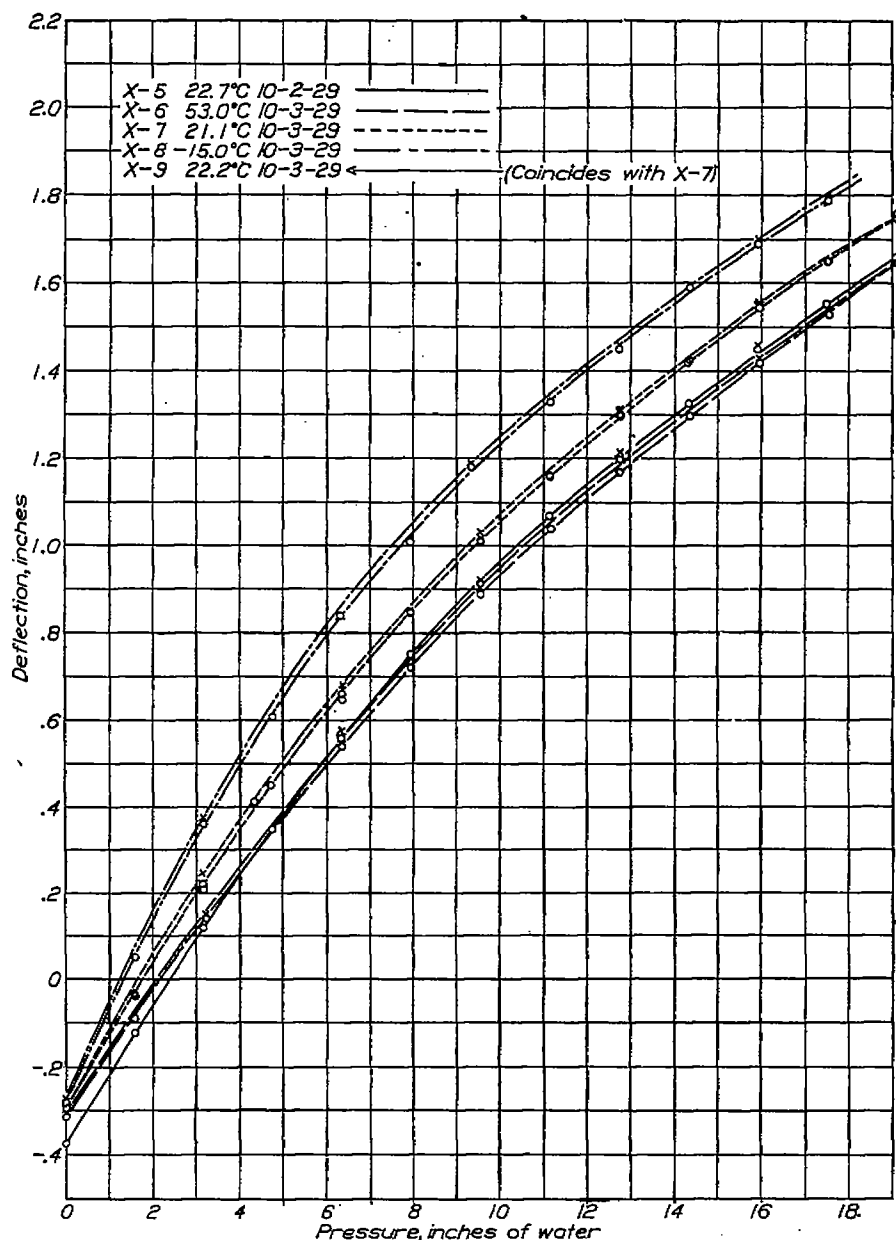

FIGURE 6.—Air-speed calibration. Instrument No. 149. Capsule No. G-280 (upper)

temperature coefficient of the moduli of elasticity has, on the other hand, not been observed directly. The conclusion arrived at in this investigation is that the stiffness is *critically dependent upon small irregularities* in the diaphragm proper, and on the *relative expansion of the diaphragm to the enclosure*, when clamped.

LIMITATIONS OF THE CLASSICAL FORMULA FOR DEFLECTION OF THIN DIAPHRAGMS

In order to emphasize the condition referred to above we will subject equation (1) to a closer study. In the

slope of the curve at zero pressure, independent of any deviation that may appear at larger pressures.

By differentiation of equation (1), we obtain as "stiffness" at zero pressure, the expression:

$$S = \frac{dP}{dW} = \frac{16}{3} \frac{E}{1-\delta^2} \frac{t^3}{r^4} \tag{2}$$

Examination of the calibration curves of Figures 2 to 6 will disclose the fact that the stiffness S is about doubled, as the temperature is increased from −15° to +53° centigrade. The expression (2), on the other

hand, should be almost a constant as far as the temperature is concerned. The contradiction is explained by the fact that the formula (2) is limited to the case of the so-called inextensional strain. That is, the neutral plane of the diaphragm is assumed free from strain. (Reference 5.) The calibration curves in the Figures 2 to 6 were taken on standard N. A. C. A. instruments with stretched diaphragms. The explanation of the rather baffling temperature dependency is thus forced upon us:

It is due to changes in the initial strain in the diaphragm, which, in turn, are due to a difference in the thermal expansion of the diaphragm and the cell body.

To recapitulate: In formula (2) the following assumptions are made:

   (1) Diaphragm homogenous and isotropic.

   (2) $\frac{t}{R}$ small.

   (3) No initial strain.

   (4) Diaphragm plane.

For the pressure cell, conditions 1 and 2 are satisfied.

Condition 3 is violated, owing to relative expansion, except for one definite value of the temperature.

Condition 4 must, for practical reasons, be given some consideration. The slack 0.00125-inch diaphragm is *stiffer* than the 0.002-inch, owing to regular star-shaped buckling caused by soldering on the central bushing. (See Table II of diaphragm data.)

It has been found, from long experience with thin pressure diaphragms, that it is necessary to stretch the diaphragm at least to a certain extent. The explicit reason for this procedure is to avoid a "double zero."

Expression (2) is useful in giving the ultimate sensitivity that can be expected for given dimensions of a diaphragm. The actual thin diaphragm will, owing to violation of conditions 3 and 4, show a stiffness of up to 30 times this limiting value. (See fig. 2.) This question is vital in connection with what has been termed "sensitive cells."

THE APPROXIMATE THEORY OF STRETCHED MEMBRANES

The classical theory of the unstretched circular diaphragm clamped along its edge and subjected to a uniform fluid pressure, gives the following differential equation:

$$px^2\pi = -\pi x\frac{t^3}{6}\frac{m^2E}{m^2-1}\left(\frac{d^3y}{dx^3}+\frac{1}{x}\frac{d^2y}{dx^2}-\frac{1}{x^2}\frac{dy}{dx}\right), \text{ where } m=\frac{1}{\delta}$$

(See Fuller and Johnston, p. 484, eq. 11.) The left side represents the fluid pressure on a circular ring of radius x and the right side represents a pure shearing force at the edge of this ring. (See fig. 7.) The membrane being unstretched, there is no tangential force. The above equation admits of an exact solution:

$$y=\frac{A}{32}(x^4-2r^2x^2) \text{ where } A \text{ is a constant.}$$

If, however, we assume an initial tension T in the diaphragm, we have:

$$p\pi x^2=2\pi xtT\frac{dy}{dx}-\pi x\frac{t^3}{6}\frac{m^2E}{m^2-1}\left(\frac{d^3y}{dx^3}+\frac{1}{x}\frac{d^2y}{dx^2}-\frac{1}{x^2}\frac{dy}{dx}\right)$$

where the first right side member indicates the pressure carried by the outward tension on the ring of radius x.

The solution of this relation is very important, because it would furnish means of predicting the initial stiffness of the membrane for any arbitrary value of the initial tension. The exact solution has not been found. In order to obtain at least some information as to the tendency of the effect we will limit ourselves to the case where the initial tension may be considered small.
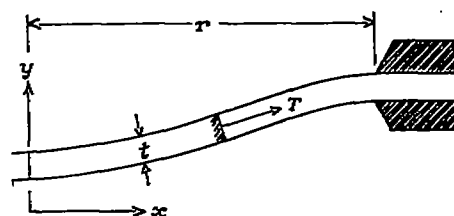
FIGURE 7

For small initial stresses we apply an approximate expression for the first right-hand member in the above equation. We have

$$y=\frac{A}{32}(x^4-2r^2x^2),\ y_m=-\frac{A}{32}r^4$$

and

$$\frac{dy}{dx}=\frac{A}{32}(4x^3-4r^2x)=-\frac{4y_m}{r^4}(x^3-r^2x)$$

which actually is correct only for zero initial stress. Replacing $\frac{dy}{dx}$ in the first right-hand member of the above equation by this approximate value, gives:

$$p\pi x^2=-2\pi xtT\frac{4y_m}{r^4}(x^3-r^2x)-\pi x\frac{t^3}{6}\frac{m^2E}{m^2-1}\left(\frac{d^3y}{dx^3}+\right.$$
$$\left.+\frac{1}{x}\frac{d^2y}{dx^2}-\frac{1}{x^2}\frac{dy}{dx}\right).$$

The solution of this equation follows. We write

$$px=-8tT\frac{y_m}{r^4}(x^3-r^2x)-\frac{m^2E}{m^2-1}\frac{t^3}{6}\left(\frac{d^3y}{dx^3}+\frac{1}{x}\frac{d^2y}{dx^2}-\frac{1}{x^3}\frac{dy}{dx}\right)$$

or $$px=L(x^3-r^2x)+M\left(\frac{d^3y}{dx^3}+\frac{1}{x}\frac{d^2y}{dx^2}-\frac{1}{x^2}\frac{dy}{dx}\right)$$

where

$$L=-8tT\frac{y_m}{r^4} \text{ and } M=-\frac{m^2E}{m^2-1}\frac{t^3}{6}.$$

Rearranging:

$$(p+Lr^2)x=Lx^3+M\left(y'''+\frac{1}{x}y''-\frac{1}{x^2}y'\right).$$

Integrating:

$$(p+Lr^3)\frac{x^2}{2}=L\frac{x^4}{4}+M\left(y''+\frac{1}{x}y'\right)+C.$$

Multiplying by x:

$$(p+Lr^2)\frac{x^3}{2}=L\frac{x^5}{4}+M(xy''+y')+Cx.$$

Integrating:

$$(p+Lr^2)\frac{x^4}{8}=L\frac{x^6}{24}+M(xy')+C\frac{x^2}{2}+D,\text{ (but }D=0).$$

Dividing by x:

$$(p+Lr^2)\frac{x^3}{8}=L\frac{x^5}{24}+My'+C\frac{x}{2}.$$

To determine C we have that $y'=0$ when $x=r$:

$$(p+Lr^2)\frac{r^3}{8}=L\frac{r^5}{24}+C\frac{r}{2}.$$

$$C=(p+Lr^2)\frac{r^2}{4}-L\frac{r^4}{12}=p\frac{r^2}{4}+L\frac{r^4}{6}.$$

Replacing C:

$$(p+Lr^2)\frac{x^3}{8}=L\frac{x^5}{24}+My'+\left(p\frac{r^2}{4}+L\frac{r^4}{6}\right)\frac{x}{2}.$$

Rearranging and integrating:

$$My=-L\frac{x^6}{144}-\left(p\frac{r^2}{4}+L\frac{r^4}{6}\right)\frac{x^2}{4}+(p+Lr^2)\frac{x^4}{32}+E\quad(E=0)$$

We obtain y_m by putting $x=r$ and further reintroduce $L=-8tT\frac{y_m}{r^4}$

$$My_m=8tT\frac{y_m}{r^4}\frac{r^6}{144}-\left(p\frac{r^2}{4}-8tT\frac{y_m}{r^4}\frac{r^4}{6}\right)\frac{r^2}{4}$$
$$+\left(p-8tT\frac{y_m}{r^4}r^2\right)\frac{r^4}{32}$$

$$y_m=\frac{p\frac{r^4}{16}+p\frac{r^4}{32}}{M-8tT\left(\frac{r^2}{144}+\frac{r^2}{24}-\frac{r^2}{32}\right)}=\frac{-p\frac{r^4}{32}}{M-8tT\frac{5r^2}{288}}=$$

$$\frac{p\frac{r^4}{32}}{-M+tT\frac{5r^2}{36}}$$

With

$$M=-\frac{m^2E}{m^2-1}\frac{t^3}{6},$$

$$y_m=\frac{p\frac{r^4}{32}}{\frac{m^2E}{m^2-1}\frac{t^3}{6}+tT\frac{5}{36}r^2}=\frac{3}{16}\frac{r^4}{t^3}\frac{p}{\frac{m^2E}{m^2-1}+T\frac{5}{6}\left(\frac{r}{t}\right)^2}\quad\text{or}$$

$$p=y_m\frac{16}{3}\frac{t^3}{r^4}\frac{m^2E}{m^2-1}\left(1+\frac{m^2-1}{m^2E}T\frac{5}{6}\left(\frac{r}{t}\right)^2\right)$$

Initial stiffness $\frac{dp}{dy_m}$ at the center equals:

$$S=\frac{16}{3}\frac{t^3}{r^4}\frac{m^2E}{m^2-1}\left(1+\frac{m^2-1}{m^2E}T\frac{5}{6}\left(\frac{r}{t}\right)^2\right)$$

that is, the original stiffness as given by formula (2) appears to have been magnified by the factor:

$$1+\frac{m^2-1}{m^2E}T\frac{5}{6}\left(\frac{r}{t}\right)^2$$

For a slack membrane, $T=0$, and the result is identical with that given in formula (2).

This result is instructive, for it shows how the stiffness depends on the initial tension T. Now, this tension is proportional to the temperature, and we have the result that the stiffness is a linear function of the temperature, as could be expected. It will be observed that the thinnest diaphragms are the ones that show the greatest temperature effects.

The above formula gives only the initial stiffness, or stiffness at zero pressure. It is possible to derive a more general form. (See page 8 of British Aeronautical Research Committee Reports and Memoranda No. 1136.)

EXPERIMENTAL VERIFICATION OF THE CAUSE OF TEMPERATURE DEPENDENCY OF THE STIFFNESS— ADOPTED METHOD OF COMPENSATION

Following up the conclusion that the only cause capable of producing the observed large temperature coefficient was the possible difference in thermal expansivity of the diaphragm relative to the cell body, a number of experiments were resorted to. In fact, a considerable number of trials were run before this conclusion was definitely accepted as being correct. In the Figures 2 to 6, it is noticed that the temperature coefficient of stiffness is positive; the stiffness increases with temperature. This fact is accounted for by the assumption that the thermal expansivity of the membrane is *less* than that of the cell body. A phosphor bronze diaphragm was subsequently inserted in the steel body of the cell. The expected reversal of the temperature dependency appeared. (See fig. 8.) The coefficient is here negative, and appears to be about four times greater than in the preceding case.

The author then adopted the procedure of employing a diaphragm composed of two metals with different coefficients of expansion, and so arranged as to give the diaphragm, as a whole, the same expansivity as the supporting body. The pressure diaphragms already carried a $\frac{5}{16}$-inch central steel bushing. As a first attempt this bushing was made of brass and increased to $\frac{3}{8}$ inch in diameter. The result of the calibration is shown in Figure 9. The instrument is overcompensated. The next trial was made with a $\frac{1}{4}$-inch brass bushing. This calibration is reproduced in Figure 10. This instrument is undercompensated to

the same degree as the preceding one was overcompensated. Consequently, a ⁵⁄₁₆-inch bushing was employed. The result is shown in Figure 11 and, with further improvements in the technique, in Figure 12. The temperature dependency is here entirely taken care of for the entire range between −15° and +53° centigrade.

was discovered that this internal friction was brought about in a different manner.

It will be seen from the schematic drawing, Figure 1, that the mirror is kept in position by a hairspring. The hairspring consisted of nine convolutions of phosphor bronze wire, the cross section of which was about 0.003 by 0.016 inch. This tiny spring adds consider-
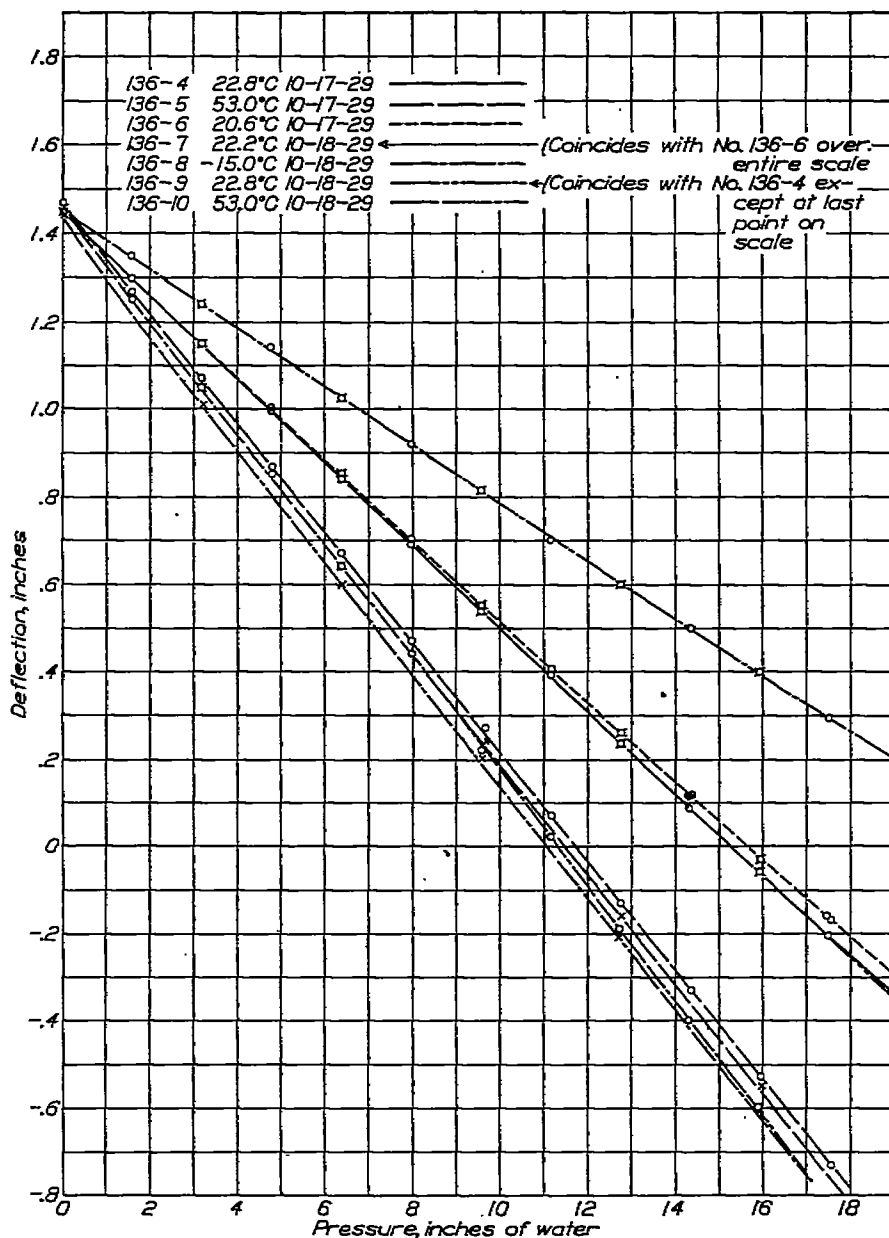

FIGURE 8.—Air-speed calibration. Instrument No. 136. Cell No. 6 (lower). 0.0035″ diaphragm with two 0.008″ copper gaskets and clamping ring. Diaphragm of phosphor bronze (plain). Brass bushing. Diaphragm stretched

FRICTION IN PRESSURE CELLS—AGING EFFECTS

One of the obstacles in making a good low-pressure cell of the type in question is the appearance of lag or dead motion. The 0.00125 diaphragm would show as much as ¼ inch "double zero." Curiously enough, it was found that this dead motion had nothing to do with hysteresis in the diaphragm, as might have been expected. After considerable experimental work it

able stiffness to the diaphragm-spring combination. We will indicate the numerical value of this stiffness.

By stiffness is meant the hydrostatic pressure needed to give a central deflection of the diaphragm of one unit. This is in accordance with the mathematical definition in formula (2). As is customary, the units used in the following are inches of water for the pressure and inches for the deflection. The
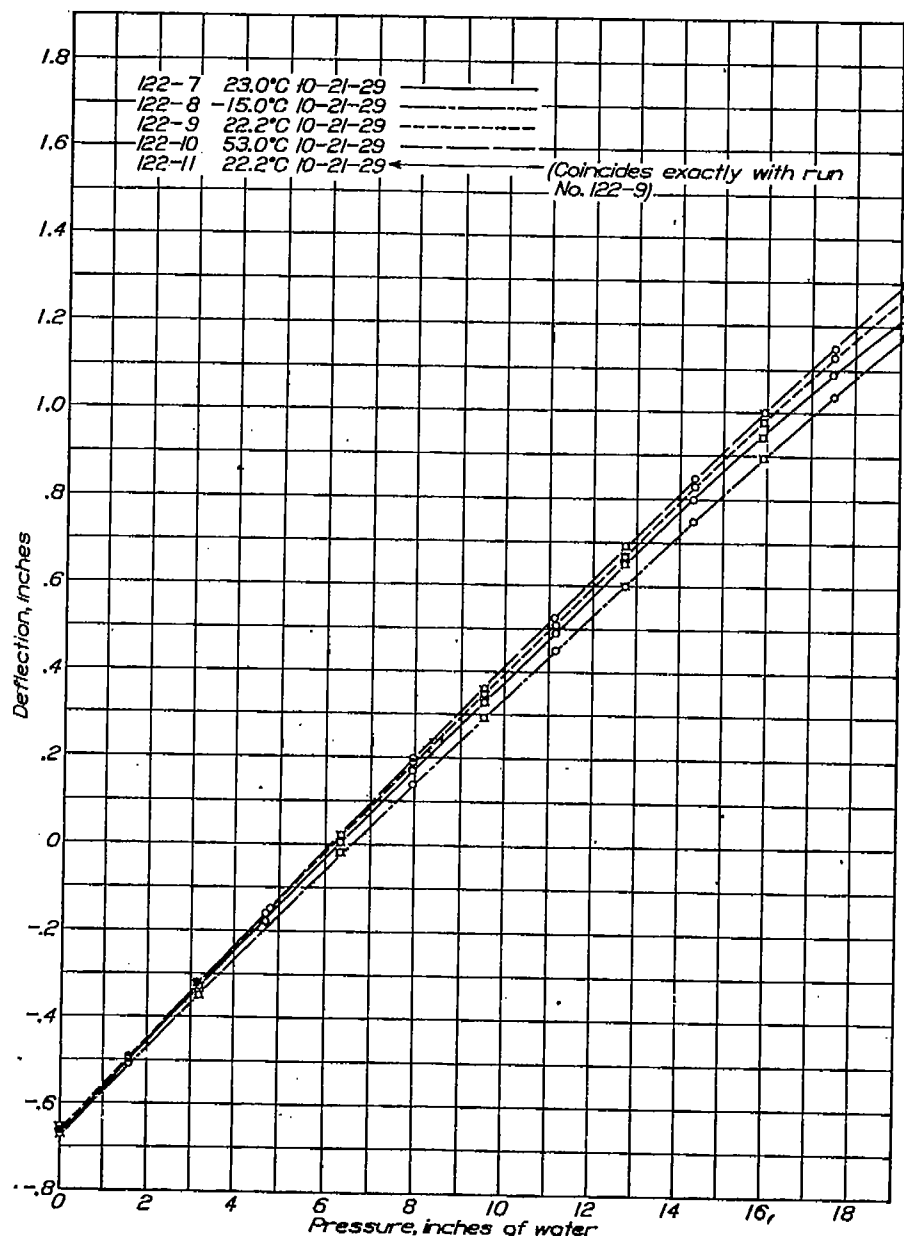
FIGURE 9.—Air-speed calibration. Instrument No. 122. Cell No. G-240 (upper). 0.002″ flat diaphragm. 0.010″ copper gaskets. ¾″ diameter brass bushing. Adjusted to 20″ H₂O. Stretched with heater. Overcompensated
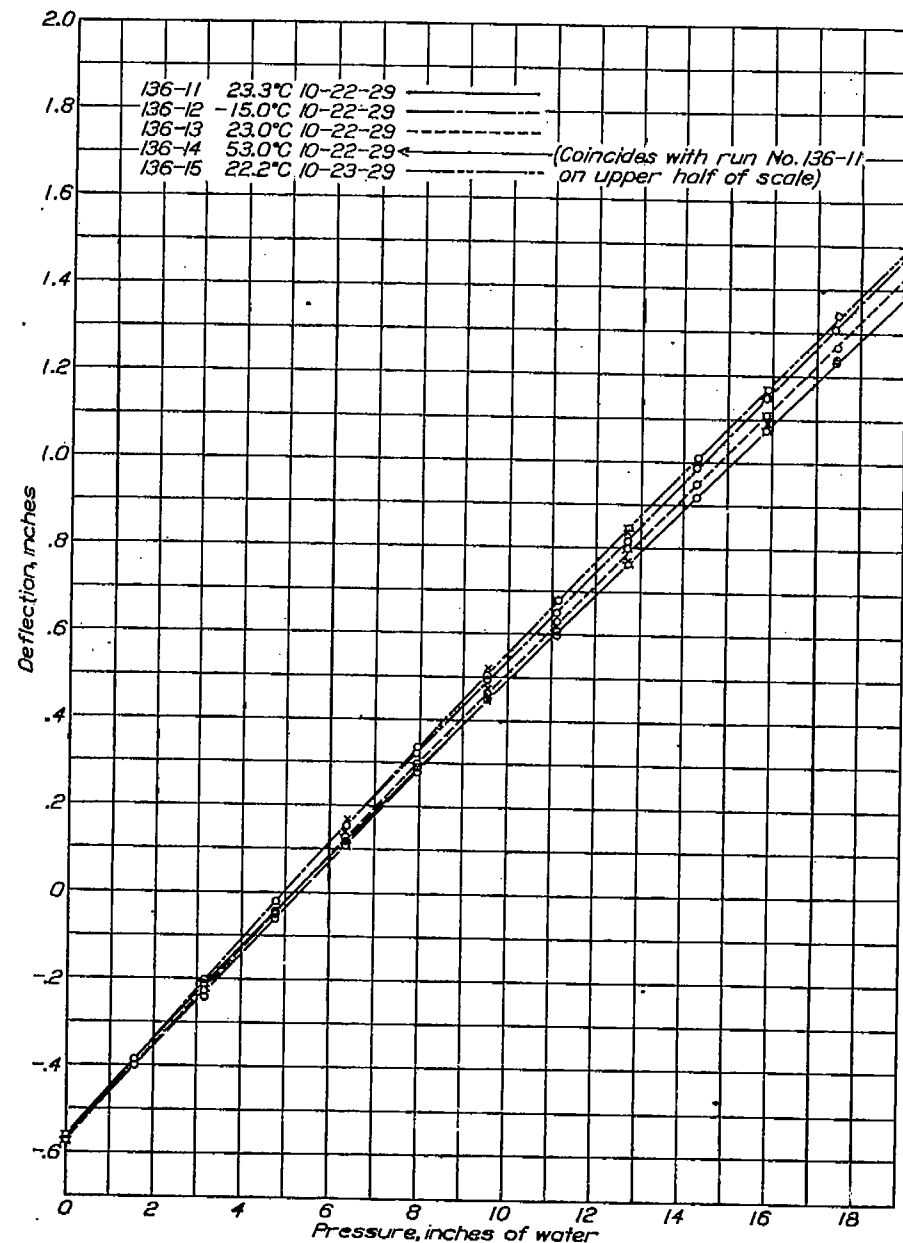
FIGURE 10.—Air-speed calibration. Instrument No. 136. Cell No. 230 (upper). 0.002″ diaphragm. Brass bushing, ¼″ flange. Two 0.006″ copper gaskets. Stretched with heater. Undercompensated
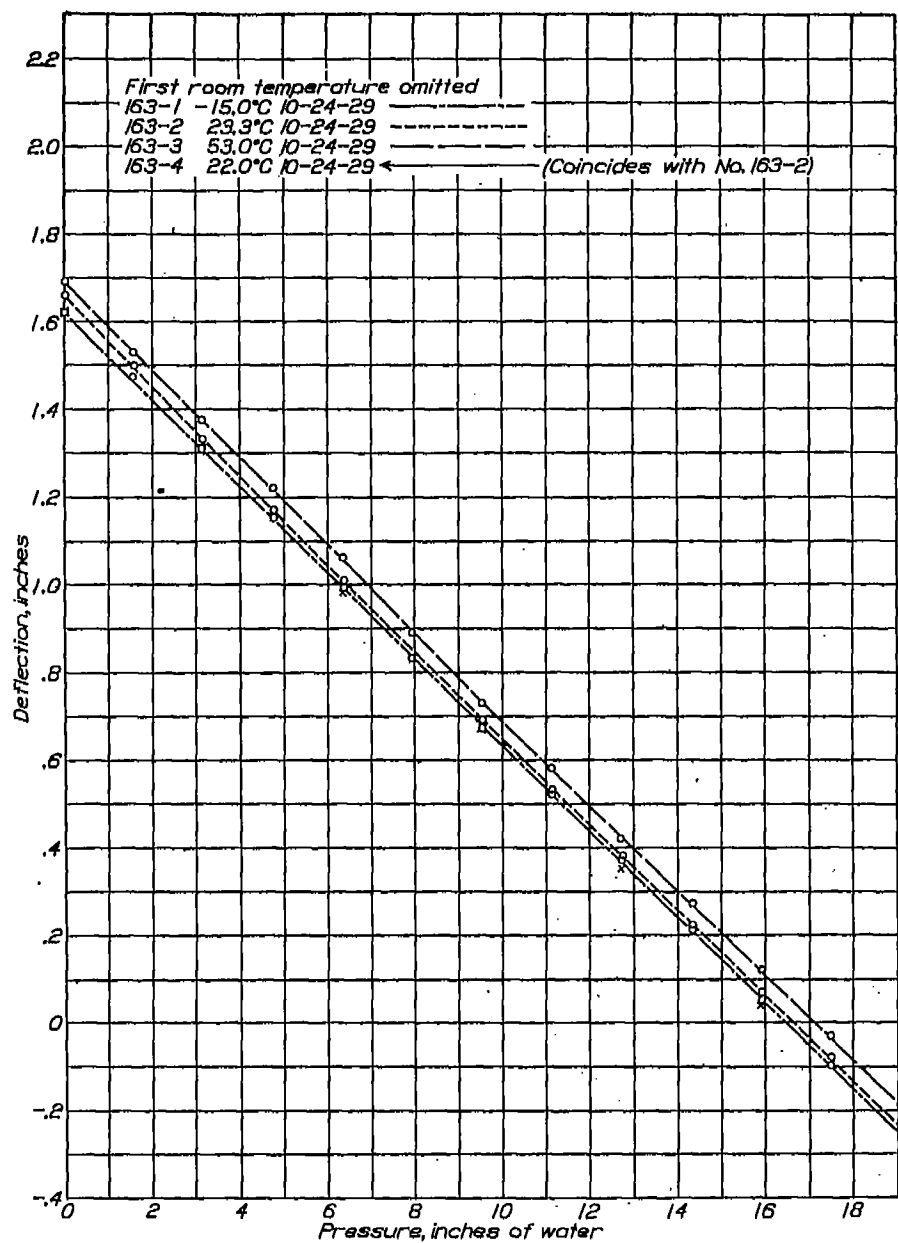
Figure 11.—Air-speed calibration. Instrument No. 163. Cell No. 6. 0.002″ diaphragm. Brass bushing with ⁵⁄₁₆″ flange. Two 0.008″ copper gaskets. Stretched with heater
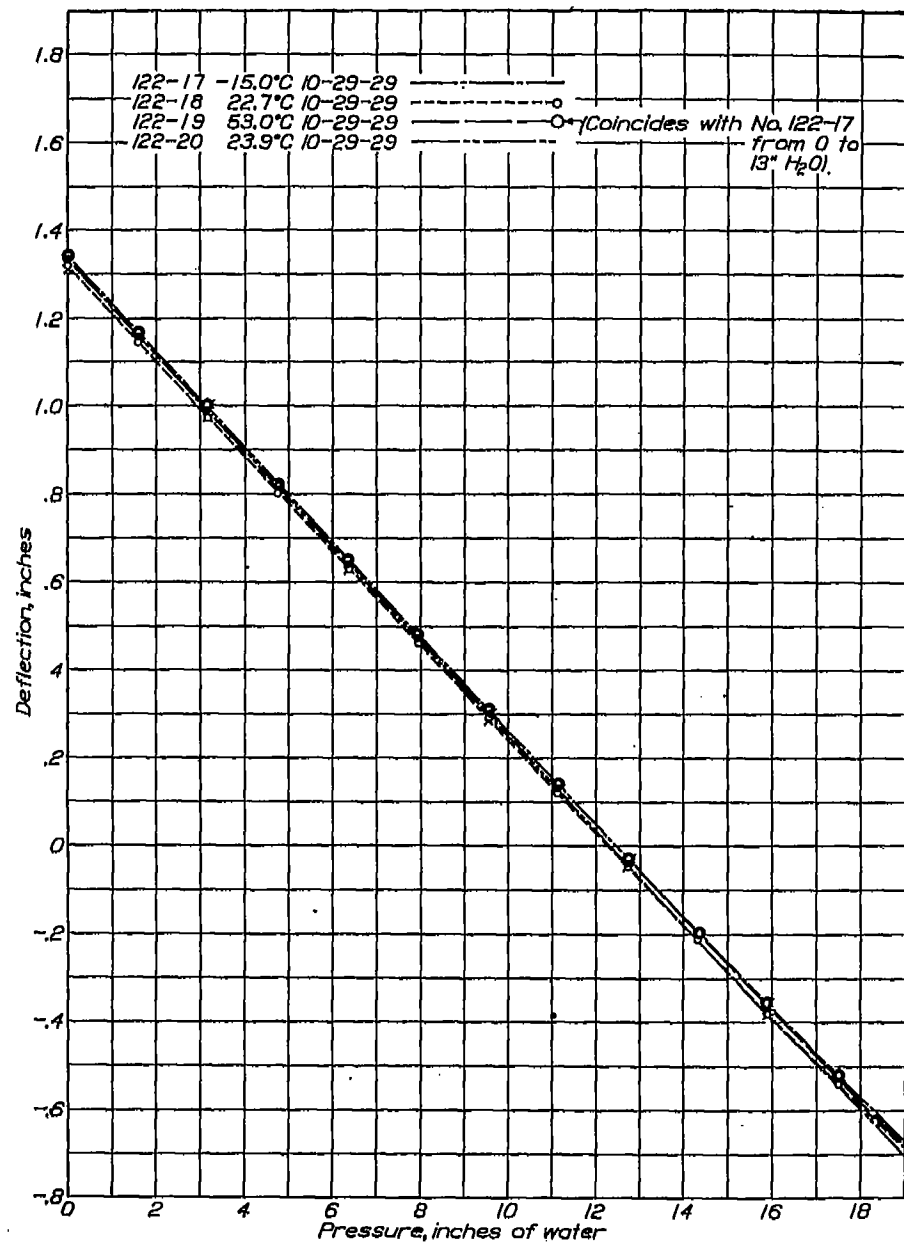

Figure 12.—Air-speed calibration. Instrument No. 122. Cell No. 280 (lower). 0.002″ diaphragm. ⁵⁄₁₆″ flange brass bushing. Two 0.005″ annealed copper gaskets. 20″ H₂O
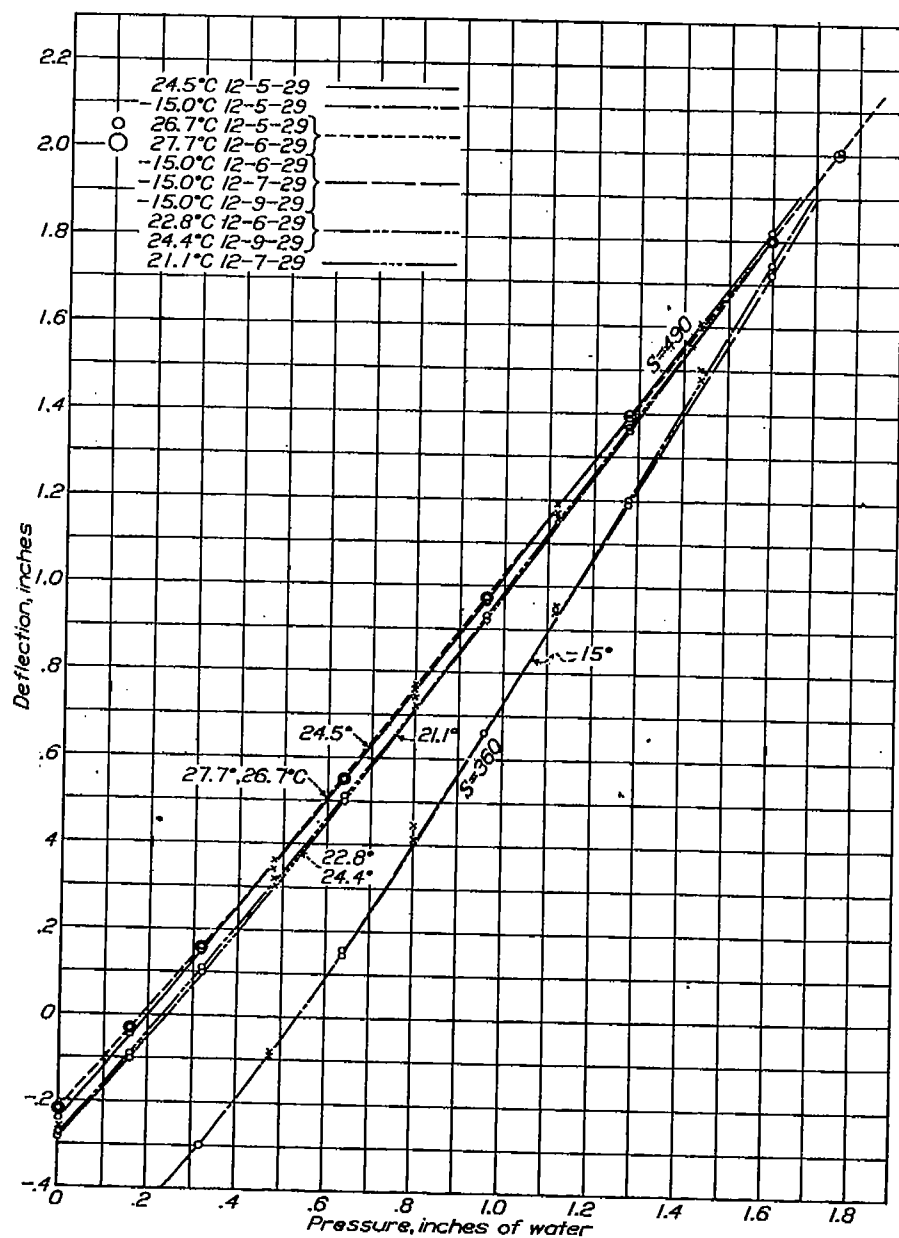
FIGURE 13.—Air-speed calibration. Instrument No. 136. Capsule No. G–279

NOTE.—0.00125″ diaphragm with ⁵⁄₁₆″ brass bushing with rubber dam between flange of bushing and diaphragm on both sides. Two copper gaskets. Slightly stretched. Adjusted to 15.8″ of water. Center distance r_c=0.018″.
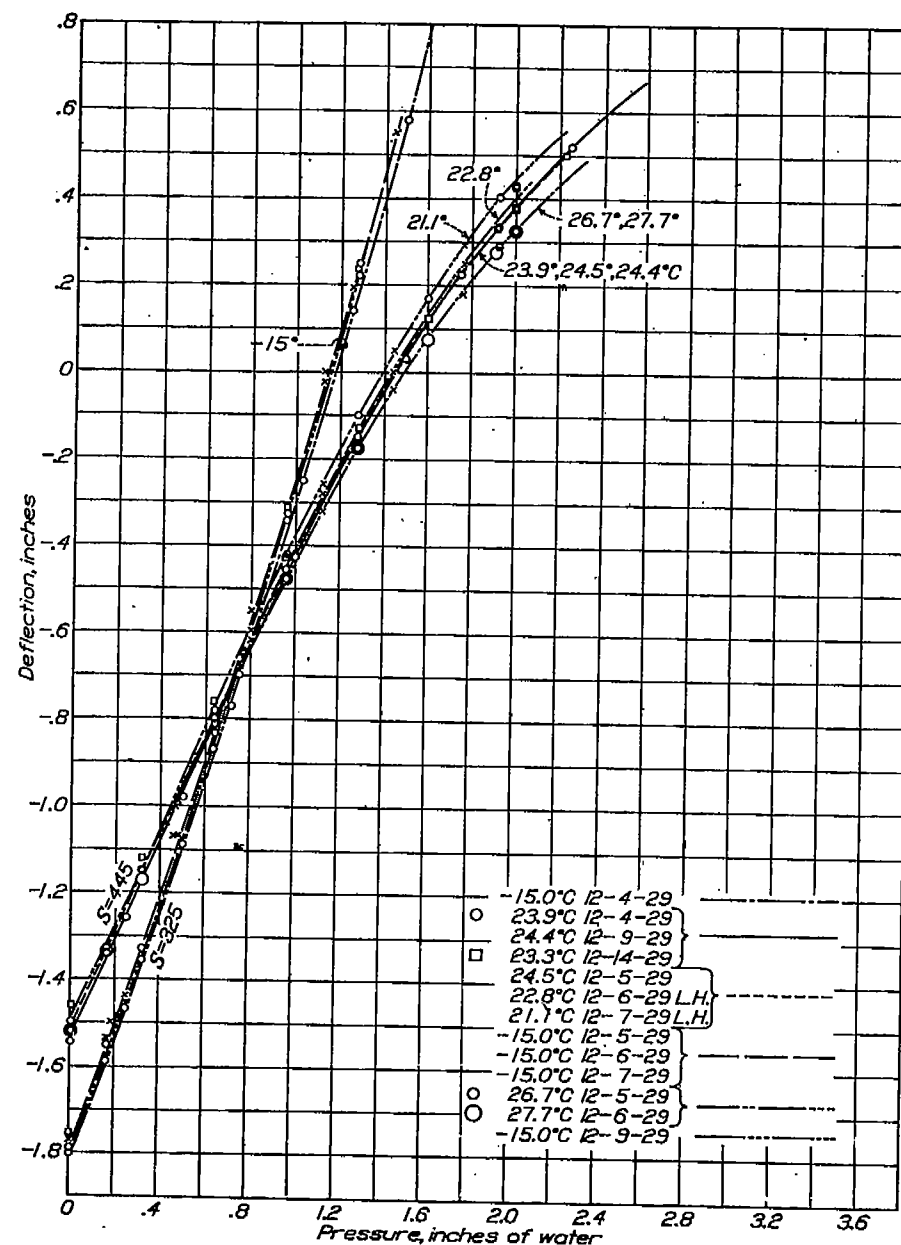
FIGURE 14.—Air-speed calibration. Instrument No. 136. Capsule No. G–230. Center distance r_c=0.020″. Large rubber gasket removed and copper gaskets installed

stiffness S accordingly is expressed as inches of water per inch deflection.

The angular stiffness of the above spring was determined directly as $M=0.190$ gram cm per radian. If the stylus is located at a distance r_0 from the axis of the spring, the latter exerts a force of $\frac{M\alpha}{r_0}$ on it, where α is the angle of angular compression of the spring. This gives a stiffness reduced to the stylus of $S_0=\frac{M}{r_0^2}$.

With the given value of M, and with r_0 equal to the adopted minimum of 0.010 inch or 0.0254 cm, we obtain:

$$S_0=\frac{0.190}{0.0254^2}=295\ \frac{g}{cm}.$$

In the limiting case of a slack diaphragm, the effect of a single central force is equivalent to four times the effect of an evenly distributed load of the same magnitude. Hence, S_0 equals 1,180 $\frac{g}{cm}$ for equivalent evenly distributed load. This value corresponds to a stiffness of 123 inches water per inch deflection. This value is high beyond expectation. A glance at the diaphragm data in Table II shows why any attempts
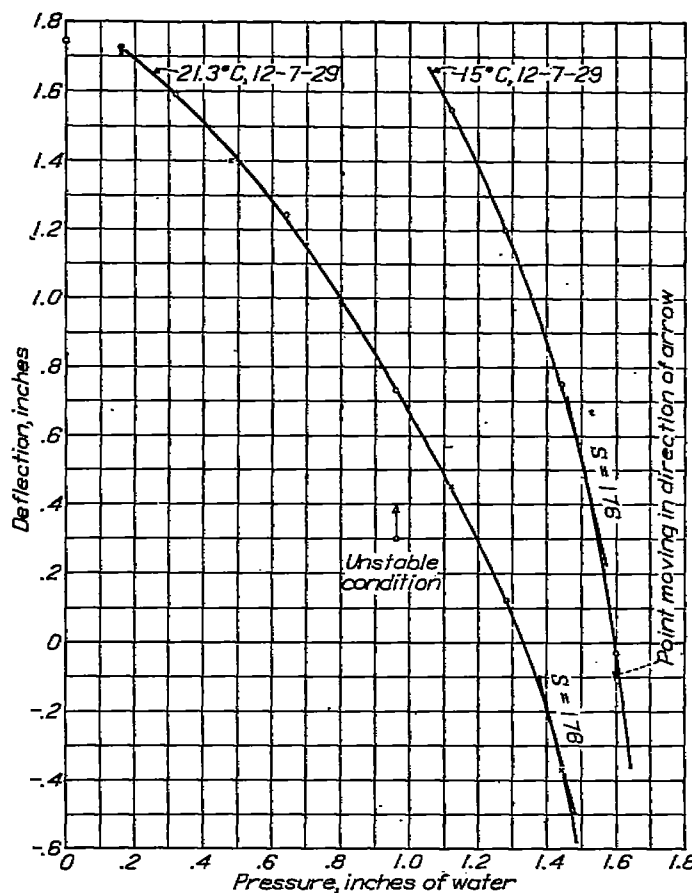
FIGURE 15.—Air-speed capsule No. 160. 0.002" diaphragm ¼" steel bushing screwed on to diaphragm. Stylus 0.013" from center. Adjusted to 1.5" of water

to use this spring with a slack diaphragm were unsuccessful.

The stiffness of the 0.00125-inch diaphragm as used amounts to only 87 inches of water per inch deflection,
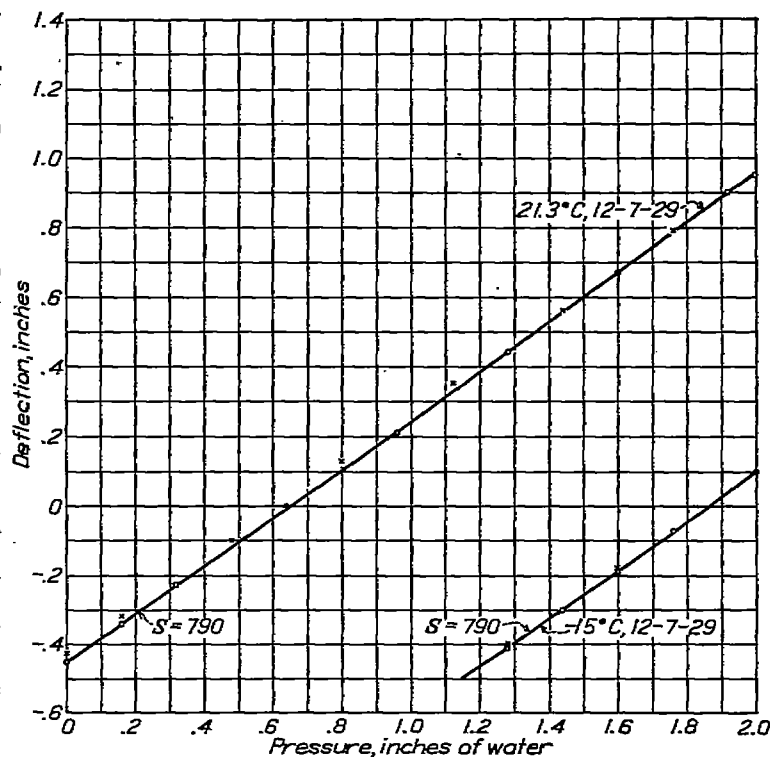
FIGURE 16.—Air-speed capsules No. G–180. 0.002" diaphragm 1¾₂" brass bushing soldered. Bushing pressed after soldering to relieve star shape of diaphragm. Adjusted to 3" of water Stylus 0.018" from center

so that in this case approximately 60 per cent of the force is transmitted through the stylus point to the back of the mirror shaft, and from there on through the bearings. This condition is highly undesirable. Not only is the effective deflection reduced, but the frictional forces reach an appreciable relative magnitude.

The spring was usually given an initial angular deflection of about 45 degrees, or somewhat less than one radian. Referring to the above value of equivalent stiffness of the spring, we find that this force is approximately equal to one inch of water. The value of the force remains fairly constant over the range of the instrument owing to this large initial deflection. This internal force on the stylus and bearings is responsible for a considerable part of the observed lag.

There is, however, another condition that causes an amplification of the lag when S is small. When the force on the stylus is large compared to the internal forces in the diaphragm, considerable lateral motion of the stylus point, relative to the mirror axis, will take place, due to bending of the diaphragm. This latter effect explains why the observed deviations always would be much larger for slack diaphragms.

As a remedy against this dead motion it was conceived that it would be necessary to employ weaker hairsprings or less initial tension than was usually employed. The last scheme was resorted to for some time. The hairspring was given an initial angular compression of only about 15°, instead of 45°. This

experiment made it quite evident that the friction was mainly due to the spring. The friction diminished, as expected, and, moreover, it became evident that the
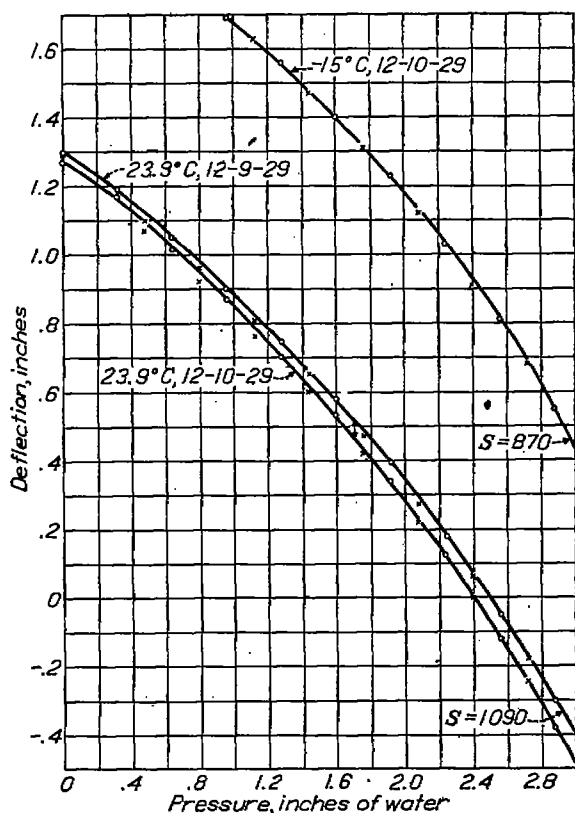

FIGURE 17.—Air-speed calibration. Instrument No. 122. Capsule No. 160. 0.002" diaphragm ¼" steel bushing with 0.010" dental rubber between flange of bushing and diaphragm on both sides. Center distance, r_s=0.011". Fiber gaskets. Stretched by blowing breath on diaphragm

magnitude of the frictional lag changed with the deflection, due to the greater proportional change in the transmitted force.

A new set of hairsprings was then obtained, which showed a stiffness of 0.055 g cm per radian, instead of the previous type of 0.190 g cm. It is on the good results obtained with these weaker hairsprings that the contention was based that the actual hysteresis in the diaphragm proper is of negligible proportions. (See fig. 18.) The quality of these instruments, as far as the hysteresis is concerned, is comparable with any of the ordinary high-pressure types. The increasing and decreasing pressure readings are marked by circles and crosses respectively. The lag is seen to be negligible.

Some instruments had a tendency to change their calibration with time. This change would be most pronounced just after the instrument was made up, but the changes would usually go on for several months. It was suspected that this effect was due to insufficient clamping of the diaphragm. It is seen from Figure 1 that the diaphragm is inserted between the cell body proper and a steel gasket. It is obvious that a slight inaccuracy in manufacture will cause the

thin diaphragm to be clamped only partially along its edge.

On introducing soft and relatively thick copper gaskets on both sides of the diaphragm instead of this single steel gasket, it was found that this disagreeable effect was removed. Two cells were calibrated at intervals for a period of more than half a year as a matter of comparison. The cell employing copper gaskets was entirely superior. This method of clamping has been adopted as standard in the N. A. C. A. pressure cells.

LOW-PRESSURE CELLS

The sensitivity of the instrument depends on:

(1) The stiffness of the diaphragm;
(2) The magnification.

As regards the magnification, the only variable element affecting it is the distance r_0 between the stylus and the mirror axis. The allowable minimum is largely a function of the workmanship. It was found that the instrument worked perfectly well with this distance made equal to one-hundredth of an inch. Beyond this limit, however, a lag would become evident and rapidly increase in magnitude. The magnification, defined as the ratio at the motion of the light spot on
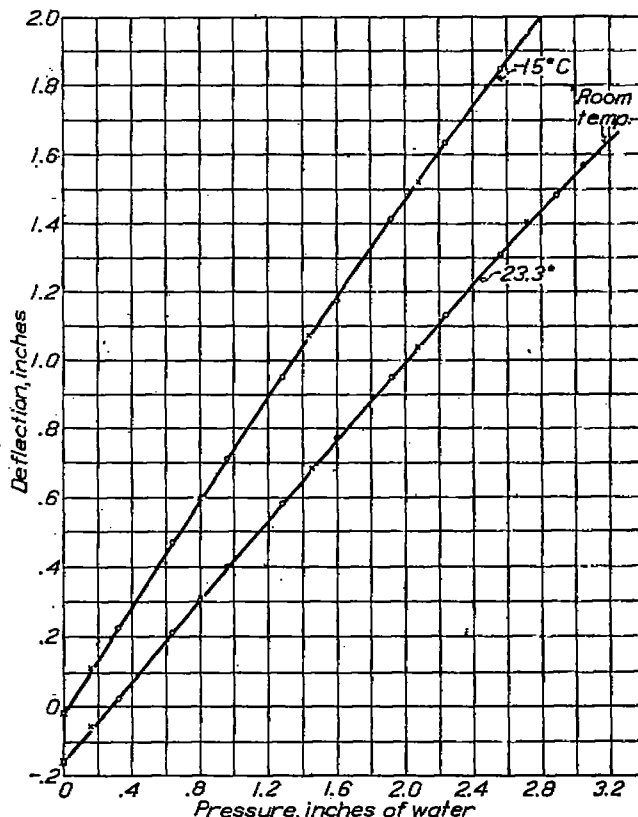

FIGURE 18.—Air-speed calibration. Instrument No. 136. Cell No. G-279. 0.00125 diaphragm with ⁵⁄₁₆" brass bushing with dental rubber between flange of bushing and diaphragm on both sides. New hairspring. Stretched with heating iron. Adjusted to 3¾" of water

the film to the equivalent deflection of the diaphragm center, is given for the N. A. C. A. instruments in Table I.

Regarding the actually observed and the theoretically expected stiffness (see fig. 20 and Table II), Figure 20 is very instructive. Curve 1 represents the well-known formula for a slack diaphragm. Curve 2 shows the actually obtained stiffness given as an average of a great number of trials. The diaphragm was inserted with care in a perfectly slack condition. The direct effect of the soldered center bushing is given by the third column of Table II and is, in itself, not very large. For instance, the 5/16-inch bushing is theoretically responsible for only 21 per cent decrease in the deflection. It has thus nothing to do with the curiously great stiffness exhibited by the very thin diaphragms.

It will be noticed from Figure 20 that the thinnest diaphragm is not the most flexible. In fact the 0.004-inch membrane is almost as flexible as the 0.00125-inch. Previously, the author had been of the opinion that this effect was caused by small irregularities in the diaphragms and that such irregularities constituted an inherent quality of the thin diaphragm, and, as such, were beyond control. Several means were tried out in order to "soften" the diaphragms

on the theory that they might tend to minimize the effect of any radial corrugations which might be present. The stiffness persistently remained, however,
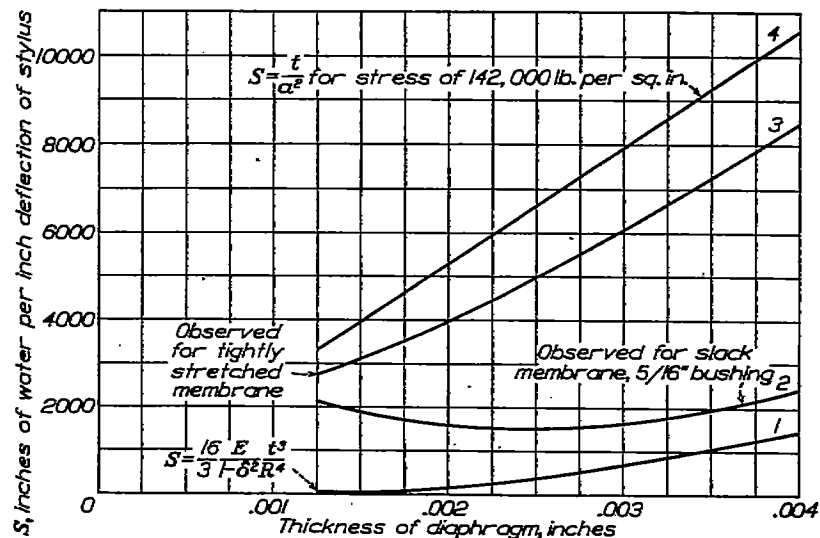

FIGURE 20.—Stiffness of the N. A. C. A. standard diaphragm sizes

NOTE.—Curve 1 shows the theoretical minimum, curve 4 the theoretical maximum of stiffness.

at not less than seven times the expected minimum value.

As a substantiation of the calculated direct effect of the center bushing, the author measured the sensitivity of a cell containing a 0.002-inch diaphragm with a very small center bushing. An increase in sensitivity of 15 per cent was expected. The result showed, however, that it was doubled. This unexpected and greatly surprising discovery furnished the explanation. The "corrugations" were not a quality of the diaphragm material, but were a necessary consequence of the method of mounting the center bushing. The bushing is soldered on. The melting point of the solder is around 300° centigrade. This means that the diaphragm is subject to considerable radial stretching in a direction toward the center, the amount being proportional to the size of the bushing and to the difference of the normal temperature and that of the melting point of the connecting metal used.

What actually happens in the case of the thin diaphragm is that it does not retain its original shape, but assumes a "star" shape. If the edge of the diaphragm is laid out on a plane it is not a straight line, but assumes a zigzag form. This result was followed up by the test of a 0.002-inch diaphragm with no center bushing. That is, the bushing still was used but it was screwed on with rubber gaskets on each side of the diaphragm, not being in any direct contact with the diaphragm proper. The calibration showed, beyond suspicion, that the contention was correct. The sensitivity was increased five times. One and one-quarter inches of water was needed for a deflection of the diaphragm of 0.00385 inch, or $S = 324$
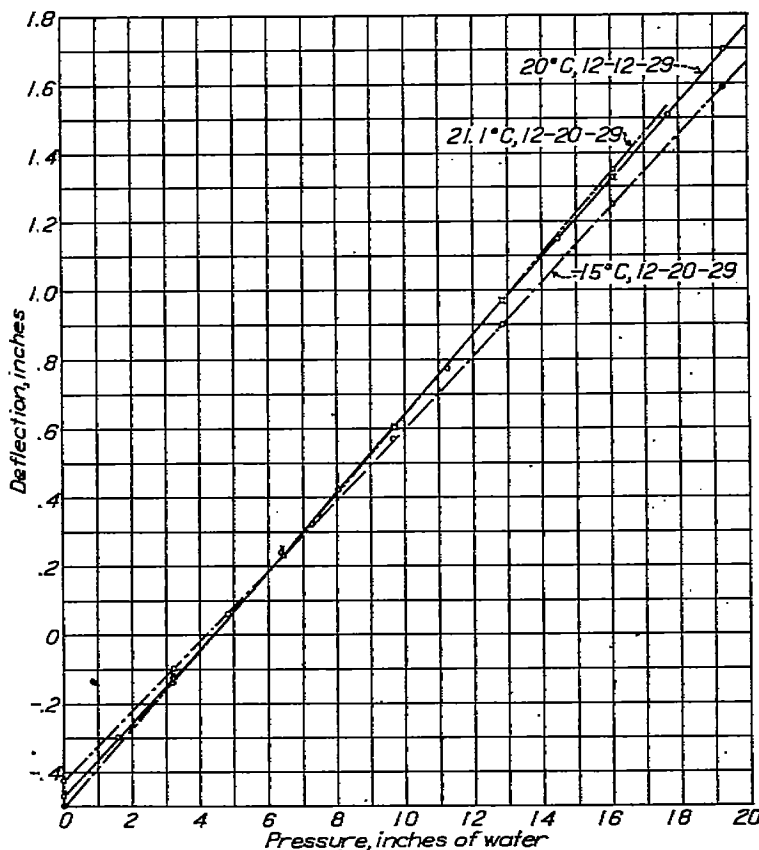

FIGURE 19.—Air-speed calibration. Instrument No. 149. Lower capsule No. G-180. High-pressure cell not properly compensated. 0.004″ diaphragm ½″ brass bushing. Two copper gaskets. Stylus 0.012″ from center. Adjusted to 18″ of water. Full stretched

without much success. It was thought that a very good initial stretching might tend to soften the diaphragm. Also, circumferential corrugations were tried

inches of water per inch deflection, which value is much closer to the theoretical minimum of 176. (See fig. 20.)

CONCLUSIONS

In this investigation, it is shown that pressure cells used for pressures above 3 to 4 inches of water can be made independent of temperature effects for all practical purposes. The serveral factors affecting the accuracy of pressure cells, namely, temperature dependency, aging effect, internal hysteresis, magnification effects, and the physical properties of thin diaphragms, have been separately studied and methods of compensation devised. The production of good pressure cells for pressures below 3 to 4 inches of water is still, however, a matter of considerable mechanical difficulty.

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  NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
  LANGLEY FIELD, VA., *February 3, 1931.*

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TABLE I

TABLE OF MAGNIFICATION OF THE N. A. C. A. STANDARD PRESSURE CELL

[Table gives the ratio of the travel of the light beam on the film to the actual displacement of the stylus]

Center distance r_1 (in inches)	—0.010	0.015	0.020	0.030	0.000
2-cell instrument	1,000	667	500	333	167
1-cell instrument	1,200	840	630	420	210
3-inch distance as used in multiple manometers	605	400	300	200	100

TABLE II

DIAPHRAGM DATA FOR THE N. A. C. A. PRESSURE CELL

Thickness of diaphragm	Theoretical stiffness at zero deflection—edge clamped		Observed stiffness at zero deflection				Approximate range of use	Theoretical stiffness at zero for a stretching of 140,000 pounds per square inch, bending neglected
	No central bushing	Soldered $\frac{9}{16}$-inch brass bushing	Slack		Stretched with heater as much as practicable; $\frac{9}{16}$-inch brass bushing soldered			
			Bushing with rubber packing	Soldered $\frac{9}{16}$-inch brass bushing				
Inches	Inch H_2O per inch	Inch H_2O per inch	Inch H_2O per inch	Inch H_2O per inch	Inch H_2O per inch	Inches H_2O	Inches H_2O per inch
0.00125	69	87	300–400	2,150	2,730	0–5	3,300
0.002	176	213	200–400	1,600	4,000	0–30	5,300
0.004	1,410	1,780		2,430	8,500	0–200	10,600

Material of diaphragm: Spring steel.
Unsupported diameter of diaphragm=1¾ inches.
Greatest deflection of diaphragm=0.01 inches.